

The thermal non-equilibrium porous media modelling for CFD study of woven wire matrix of a Stirling regenerator



S.C. Costa^{a,*}, I. Barreno^a, M. Tutar^{b,c}, J.A. Esnaola^b, H. Barrutia^b

^a CS Centro Stirling S. Coop, Avda. Alaba 3, 20550 Aretxabaleta, Spain

^b Mechanical and Manufacturing Department, Engineering Faculty of Mondragon University, Loramendi 4, 20500 Mondragon, Spain

^c IKERBASQUE, Basque Foundation for Science, 48011 Bilbao, Spain

ARTICLE INFO

Article history:

Received 5 May 2014

Accepted 8 October 2014

Keywords:

Stirling regenerator

Porous media

Heat transfer

Pressure drop

CFD

ABSTRACT

Different numerical methods can be applied to the analysis of the flow through the Stirling engine regenerator. One growing approach is to model the regenerator as porous medium to simulate and design the full Stirling engine in three-dimensional (3-D) manner. In general, the friction resistance coefficients and heat transfer coefficient are experimentally obtained to describe the flow and thermal non-equilibrium through a porous medium. A finite volume method (FVM) based non-thermal equilibrium porous media modelling approach characterizing the fluid flow and heat transfer in a representative small detailed flow domain of the woven wire matrix is proposed here to obtain the porous media coefficients without further requirement of experimental studies. The results are considered to be equivalent to those obtained from the detailed woven wire matrix for the pressure drop and heat transfer. Once the equivalence between the models is verified, this approach is extended to model oscillating regeneration cycles through a full size regenerator porous media for two different woven wire matrix configurations of stacked and wound types. The results suggest that the numerical modelling approach proposed here can be applied with confidence to model the regenerator as a porous media in the multi-dimensional (multi-D) simulations of Stirling engines.

© 2014 Elsevier Ltd. All rights reserved.

1. Introduction

The Stirling engine is an attractive alternative machine for micro-cogeneration mainly for the achievable high efficiencies and the possibility to use different heat sources. Recent studies on Stirling engines are motivated by the need to reduce the energy consumption, to improve energy efficiency and to use clean energy technologies. Despite of having many theoretical advantages, Stirling engines still face some challenges in terms of efficiency and the design process.

The recent Stirling engines studies mainly focus on the multi-dimensional (multi-D) analysis [1–7], which is based on full Stirling engine modelling, and can reproduce useful Stirling engine performance results in a time-frame short enough to impact design decisions [3]. In the multi-D analysis, the regenerator is a difficult component to be modelled because any numerical inaccuracies will influence the full scale Stirling simulation. In the modelling of a regenerator, an important factor is the internal geometry of the matrix and most of the regenerator models do not assume a

precise geometrical shape for the elements of the regenerator. In the Stirling engine's multi-D analysis the regenerator is usually modelled as a macro-scale porous medium, the porous media model requires the input of a friction factor and heat transfer coefficient which mostly are empirically obtained.

There are several works on modelling of Stirling regenerator as a porous media [8–16]. Cleveland State University [8] worked on CFD to develop a multi-D computer model of a portion of the regenerator matrix. Park et al. [9] indicated that screen-laminate matrices can be modelled as a porous media using a local thermal non-equilibrium model. Kim [10] studied the flow friction associated with laminar pulsating flows through porous media mainly for understanding the Stirling regenerators and pulse tube cryocoolers. Tao et al. [11] investigated the fluid flow and heat transfer performances of mesh regenerators under different mesh geometric parameters and material properties based on an anisotropic porous media. Landrum et al. [12] modelled porous media hydrodynamic parameters adjusting iteratively to match the model predictions to the experimental results. Conrad et al. [13] determined the hydrodynamic parameters of stacked discs matrices using a 2-D CFD assisted methodology, whereby the hydrodynamic resistance parameters for these fillers are specified when they are modelled

* Corresponding author. Tel.: +34 943 037 948; fax: +34 943 792 393.

E-mail address: ccosta@centrostirling.com (S.C. Costa).

Nomenclature

A_{fs}	interfacial area density (m^2)	S_i	source term (Pa/m)
a_1, a_2	friction factor correlation equation constants (–)	$S_{f,s}^h$	fluid/Solid enthalpy source (W/m^3)
$\vec{B}_f$	body forces (Pa/m)	s	cell size (m)
C_2	inertial resistance factor (–)	T_f	working gas temperature (K)
C_E	Ergun coefficient (–)	T_s	solid medium temperature (K)
C_f	friction factor (–)	t	time (s)
d_h	regenerator matrix hydraulic diameter (m)	u	inlet velocity magnitude in x direction (m/s)
d_w	wire diameter (m)	u_0	amplitude of oscillating inlet velocity (m/s)
E_f	total fluid energy (J/kg)	$u_{\max}$	matrix velocity magnitude in x direction (m/s)
E_s	total solid energy (J/kg)	$\vec{v}$	overall velocity vector (m/s)
f	frequency (Hz)	α	intrinsic permeability (m^2)
h	heat transfer coefficient ($\text{W/m}^2 \text{K}$)	β	Forchheimer coefficient ($1/\text{m}$)
h_{fs}	heat transfer coefficient for fluid/solid interface ($\text{W/m}^2 \text{K}$)	μ	fluid dynamic viscosity (kg/m s)
k_f	working gas thermal conductivity (W/m K)	ρ_f	fluid density (kg/m^3)
k_s	solid thermal conductivity ($\text{W/m}^2 \text{K}$)	ρ_s	solid density (kg/m^3)
L	length of regenerator matrix (m)	Π_v	regenerator matrix volumetric porosity (–)
Nu	Nusselt number (–)	φ	regenerator matrix specific heat transfer area ($1/\text{m}$)
p	static pressure (Pa)	γ	porosity of the media (–)
∇p	pressure gradient (Pa/m)	τ	non-dimensional time (–)
Re	Reynolds number (–)	$\vec{\tau}$	stress tensor (Pa)

as anisotropic porous media. Nair and Krishnakumar [14] used a 2-D local thermal equilibrium porous media model to investigate the heat transfer in a wire screen mesh regenerator. Forooghi et al. [15] numerically investigated the steady and pulsatile flow and heat transfer in a channel lined with two porous layers subject to constant wall heat flux under local thermal non-equilibrium conditions. Pathak [16] carried out a systematic experimental and CFD-based study for the quantification of Darcy permeability and Forchheimer coefficients for porous structures under steady and periodic flow conditions.

The aim of this study is to present a stepwise numerical modelling approach to obtain the inputs required (viscous/inertia resistance and heat transfer coefficient) for accurate modelling of the regenerator as a porous media (Fig. 1).

2. Numerical modelling study

2.1. Previous numerical studies: validation

The main physical phenomena to be considered in the characterization of the Stirling regenerator are the pressure drop and the heat transfer and the trade-off between both through the regenerator is crucial.

The first step of the numerical characterization of Stirling regenerator is the pressure drop characterization study [17]. A detailed 3-D isothermal model with an initial flow field is proposed for determining pressure drop characteristics (Fig. 2a), thus providing a straightforward insight in how to determine frictional losses at different geometric configurations in the typical range of Reynolds number for Stirling regenerators (Fig. 2b). The friction factor is calculated at local instantaneous Reynolds number.

The heat transfer characterization study [18] is the second step of the numerical characterization of woven wire regenerator under non-isothermal flow computations concerning the heat transfer effects of pressure drop phenomena. The model complexity is increased because the flow simulation is transient (time-dependent) and non-isothermal due to inclusion of heat transfer effects (Fig. 3a).

These first two steps [17,18] involve the validation of the numerical pressure drop and heat transfer characterization, through a small detailed 3-D stacked woven wire regenerator matrix with the use of well-known experimental correlations obtained under oscillating flow conditions [19]. Considering that the numerical results agree well with the empirical correlations, the approach is further extended to wound woven wire matrices to numerically derive the friction factor and the Nusselt correlation equations at local instantaneous Reynolds number [17,18].

Finally, the characterization is extended to a simplified equivalent porous media model of a representative detailed 3-D matrix in order to model a full length regenerator matrix under oscillating regeneration cycles based on local instantaneous friction factor and Nusselt number correlation equations.

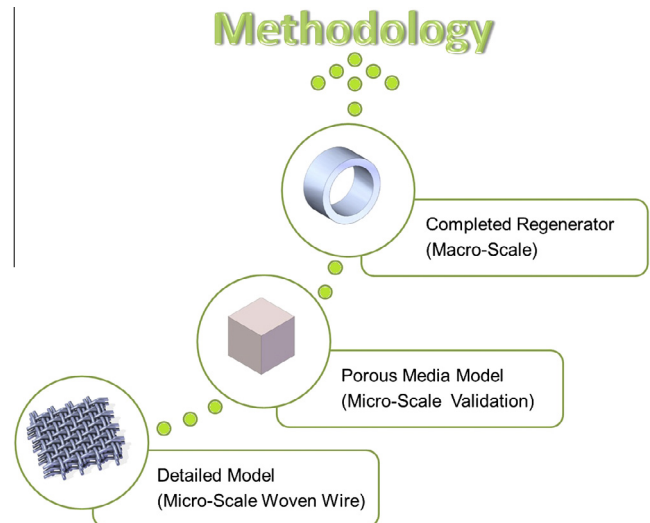


Fig. 1. Methodological approach for stirling regenerator study.

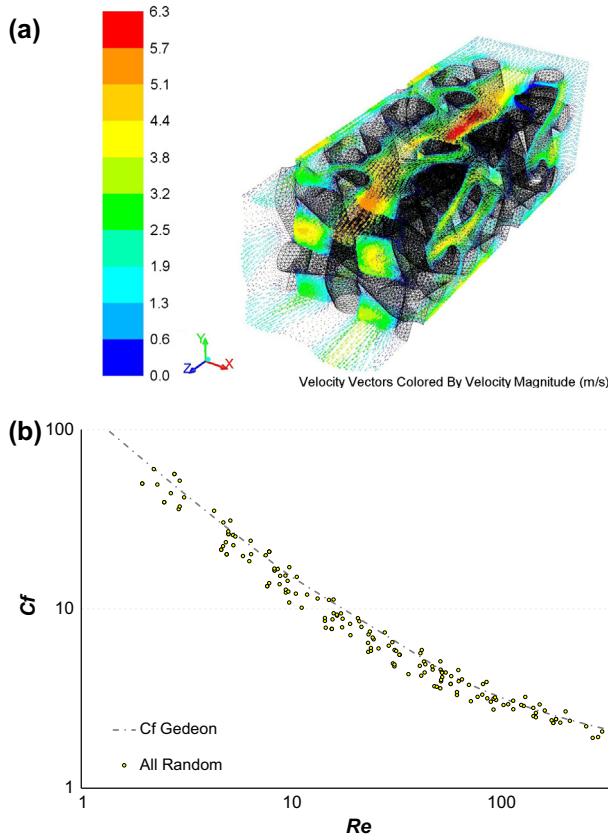


Fig. 2. Pressure drop numerical validation [22]: (a) stacked woven wire matrices: friction factor vs. Reynolds number and (b) velocity vector through stacked woven wire matrix S110 for $Re \approx 50$ (0.9 m/s).

2.2. Stirling engine regenerator modelled as porous media

The flow through a porous media is modelled by adding an extra source term to the momentum equation and another one, which accounts for heat transfer in the interface, to the energy equation depending on which thermal porous media model is employed. In general, the Darcy's law is used in porous media to describe slow or creeping flows and the Forchheimer's law is used for the description of high velocity flows.

The Darcy's law is obtained from performing steady-state unidirectional flow experiments for a uniform sand column and can be written as Eq. (1):

$$\nabla p = -\frac{\mu}{\alpha} \vec{v} \quad (1)$$

The Forchheimer's law is based on Darcy's law including the inertial effects. The inertial effects start to dominate the flow through porous media in the high velocity regime. The Forchheimer equation is given as follows:

$$\nabla p = -\frac{\mu}{\alpha} \vec{v} - \beta \rho_f |\vec{v}| v_i \quad (2)$$

The Darcy's law and the Forchheimer's law coefficients are both commonly obtained experimentally in order to describe the flow through a porous medium. Here both coefficients are obtained based on previous validated numerical results through a small detailed 3-D woven wire matrix domain as a representative of the whole Stirling regenerator [17]. The micro scale model includes individual wires trying to catch any non-uniformity of 3-D flow inside the matrix as it is shown in Fig. 2a.

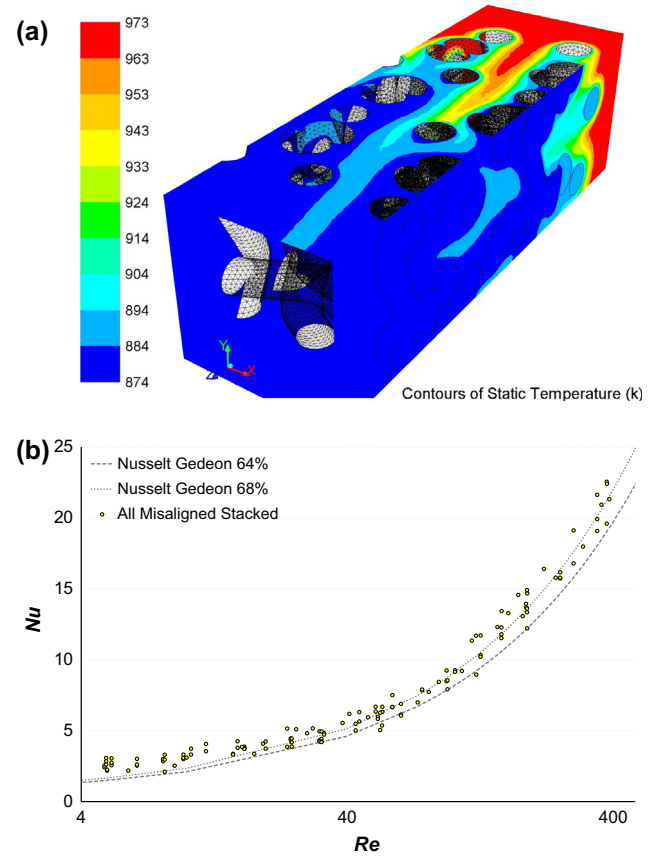


Fig. 3. Heat transfer numerical validation [23]: (a) stacked woven wire matrices: Nusselt vs. Reynolds number and (b) fluid temperature through stacked woven wire matrix S110 for $Re \approx 50$ (0.9 m/s) at 0.02 s.

Ergun [20] proposed an expression for the Forchheimer coefficient based on the experimental investigation of fluid flow through packed columns and fluidized beds:

$$\beta = \frac{C_E}{\sqrt{\alpha}} \quad (3)$$

From Eqs. (2) and (3), the Ergun equation is obtained:

$$\nabla p = -\frac{\mu}{\alpha} \vec{v} - \frac{C_E}{\sqrt{\alpha}} \cdot \rho_f \cdot |\vec{v}| v_i \quad (4)$$

The Ergun coefficient is strongly dependent on the flow regime. For the slow flows, the inertial effects are very small and can be neglected. This reduces the Forchheimer equation to the Darcy equation. As the flow velocity increases, inertial effects also increase and the flow adapts to the Forchheimer flow regime. These inertial effects are accounted for by the Ergun coefficient and the kinetic energy of the fluid.

Ergun [20], also determined the general form friction factor equation and this form is used for many friction factor correlation equations in Stirling regenerators [17,21–23]:

$$C_f = \frac{a_1}{Re} + a_2 \quad (5)$$

Considering the definition of the pressure drop as follows:

$$\Delta p = C_f \frac{L}{d_h} \frac{\rho_f}{2} u_{\max}^2 \quad (6)$$

The maximum flow velocity $u_{\max}$ is obtained by dividing the frontal maximum velocity by matrix volumetric porosity. The hydraulic diameter of the regenerator matrix, d_h , is defined by Gedeon and Wood [19] for a woven wire matrix as:

$$d_h = \frac{d_w \cdot \Pi_v}{(1 - \Pi_v)} \quad (7)$$

Therefore, Reynolds number can be given as:

$$Re = \frac{\rho_f d_h \cdot u_{\max}}{\mu} \quad (8)$$

Considering one-dimension, Eq. (6) is transformed into Eq. (9) using Eqs. (5) and (8) where the coefficients of Eq. (4) can be easily obtained from two-parameters friction factor correlation equations (Eq. (5)).

$$\Delta p = \frac{\mu \cdot a_1}{2} \frac{L}{d_h^2} u_{\max} + \frac{a_2}{2} \frac{L}{d_h} \rho_f u_{\max}^2 \quad (9)$$

The above two parameters based friction factor correlation equation (Eq. (5)) is derived here to obtain the source term included into momentum equations for two specific wire woven wire matrix configurations based on the numerical results.

In the heat transfer model through a porous media, there are two possible thermodynamic scenarios: local thermal equilibrium and local thermal non-equilibrium. In the present study, the scenario is the local thermal non-equilibrium, and the heat transfer coefficient, which is required to define the source term in the energy equations, is obtained based on numerical results.

2.3. Numerical pressure drop characterization

In this section the detailed numerical results for two specific matrix configurations with a volumetric porosity of 63% and 110 μm wire diameter, S110–63% (stacked) and W110–63%

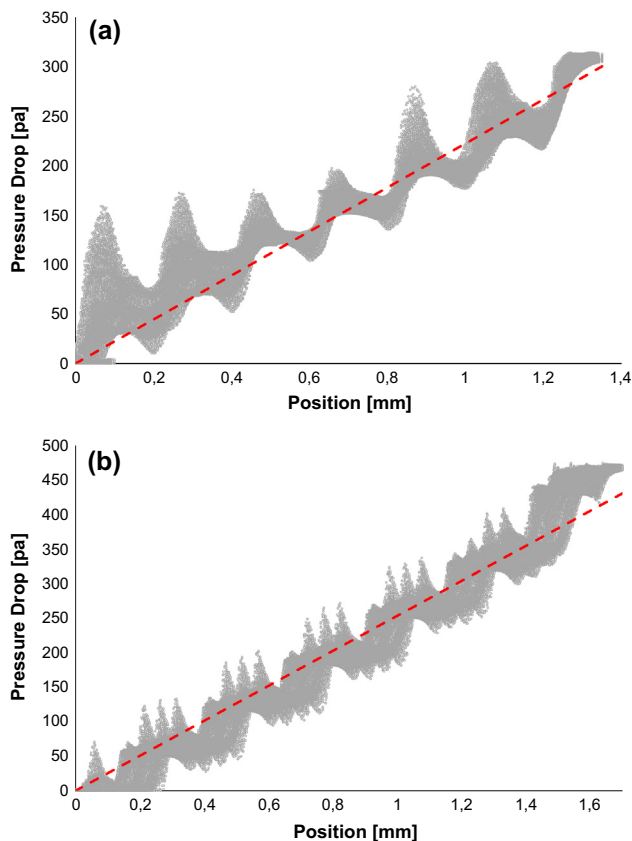


Fig. 4. Pressure drop (Pa) vs. Position (mm) through woven wire matrix of 63% volumetric porosity for $Re \approx 65$ (1 m/s): (a) stacked and (b) wound.

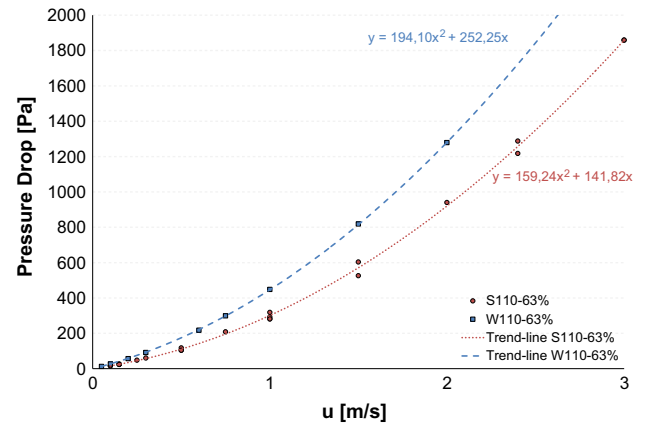


Fig. 5. Pressure drop (Pa) vs. inlet velocity (m/s) trend-line for volumetric porosity of 63% at constant working fluid properties.

(wound) are presented. These results are used as the input of the porous media model for the Stirling regenerator.

Fig. 4 shows the pressure drop results as a function of the length through the regenerator matrix, for stacked and wound configuration for $Re \approx 65$ (inflow velocity of 1 m/s). It is observed that the pressure drop behaviour is stepwise and uniform through the regenerator matrix and the trend is linear. The lower pressure drop gradient is obtained through the stacked matrix configuration.

Fig. 5 presents the plot of the pressure drop as a function of the inlet velocity through a stacked and wound matrix configuration. It is observed that the trend-line is approximated to a second order polynomial which corresponds to Forchheimer's law (Eq. (2)). These numerical results can be extrapolated to determine the coefficients for the equivalent porous media. It should be noted here that all numerical results are obtained under constant working fluid properties of density and viscosity.

Fig. 6 shows the relationship between friction factor and Reynolds number for the numerical results obtained for a stacked and wound matrix configuration. Based on the numerical results, a specific friction factor correlation equation is obtained for each matrix configuration. The specific correlation equation for S110–63% is given by Eq. (10):

$$C_f = \frac{111.8}{Re} + 1.85 \quad (10)$$

And the specific friction factor correlation equation for W110–63% is given by Eq. (11):

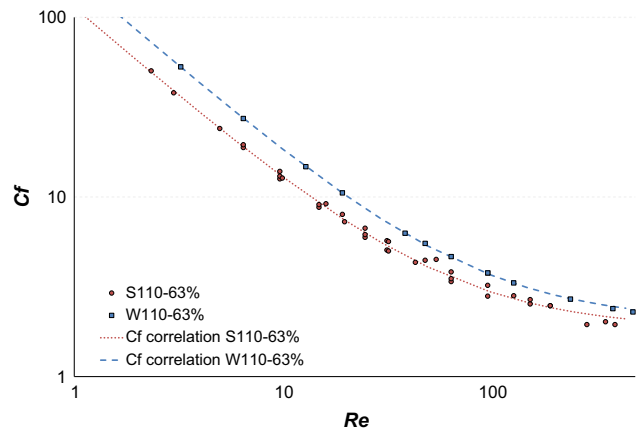


Fig. 6. Specific friction factor vs. Reynolds number for stacked and wound woven wire matrices of 63% volumetric porosity.

Table 1
S110–63% and W110–63% regenerator matrix parameters.

Matrix	Π_v (%)	ϕ (1/ μm)	d_h (μm)	L (mm)	$1/\alpha$	C_2
S110–63%	63	0.013	187	1.41	$2.47\text{E}+9$	$2.49\text{E}+4$
W110–63%	63	0.013	187	1.63	$3.72\text{E}+9$	$2.69\text{E}+4$

$$C_f = \frac{165.5}{\text{Re}} + 2.04 \quad (11)$$

Although the previously proposed three-parameters based general correlation equations by Costa et al. [17] can be used for stacked and wound woven wire matrix configurations, the above derived specific correlation equations (Eqs. (10) and (11)) can be directly used for each matrix configuration to model the porous media. In both configurations, the two-parameter Ergun form (Eq. (5)) fits very well with the numerical results and the introduction of the third parameter is not required as shown in Fig. 6.

Considering the above specific friction factor correlation equations (Eq. (10) and (11)), by rearranging Eq. (9) and making use of the relation between maximum velocity and inlet velocity, the viscous and the inertial resistance coefficients can be also obtained. Table 1 summarizes the resistance coefficients and the matrix characteristics for both configurations.

2.4. Numerical heat transfer characterization

This section presents the numerical heat transfer results obtained from the second step solutions of two specific matrix configurations, S110–63% and W110–63%. These results are used as the input of the porous media model for the Stirling regenerator.

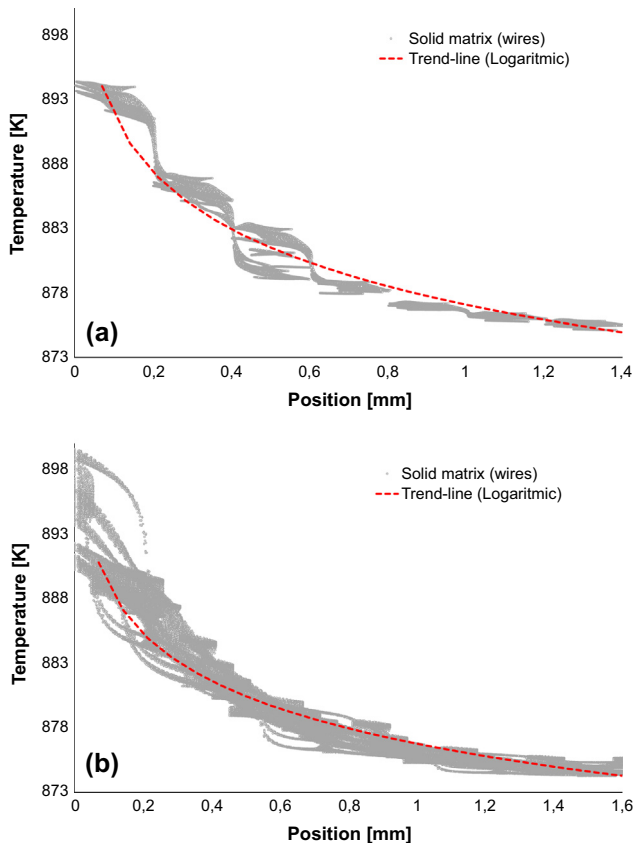


Fig. 7. Wire temperature (K) vs. Position (mm) through woven wire matrix of 63% volumetric porosity for $\text{Re} \approx 65$ (1 m/s) at 0.02s: (a) stacked and (b) wound.

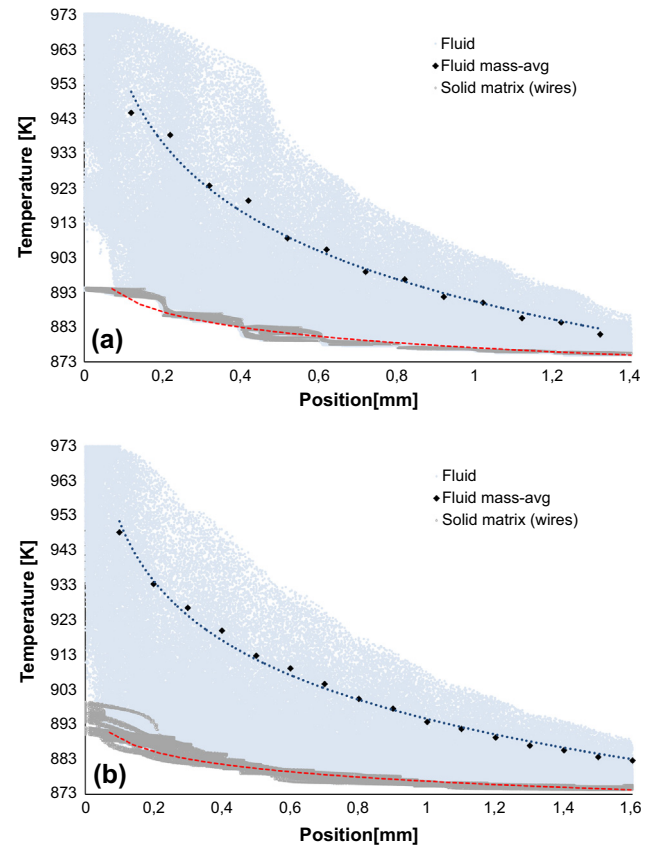


Fig. 8. Wire and working fluid temperature (K) vs. Position (mm) through woven wire matrix of 63% volumetric porosity for $\text{Re} \approx 65$ (1 m/s) at 0.02 s: (a) stacked and (b) wound.

Fig. 7 shows the wire temperature distribution as a function of the length through the regenerator matrix for a stacked and wound woven wire matrix. A clear stepwise temperature gradient is observed in the S110–63% matrix while a smooth temperature gradient mainly due to the longitudinal conduction through the wires is observed in the W110–63% matrix. However, in both cases the temperature across the regenerator matrix is not linear, and the trend-line is logarithmic.

Fig. 8 shows the wire and working fluid temperature distribution as a function of the length through the regenerator matrix for the stacked and wound woven wire matrices. The figure also

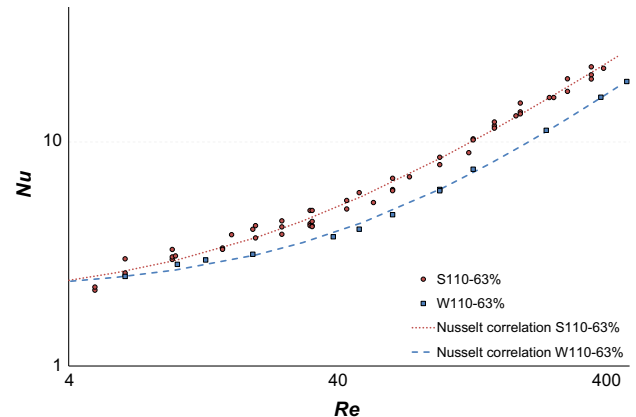


Fig. 9. Specific Nusselt number versus Reynolds number for stacked and wound woven wire matrices of 63% volumetric porosity.

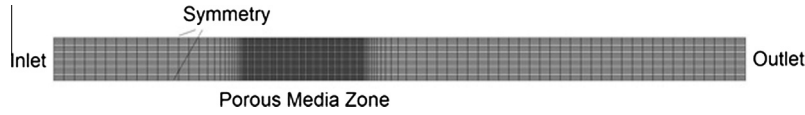


Fig. 10. Computational domain for equivalent porous media model.

plots the trend-line for the wire temperature and for the mass-average working fluid, it can be appreciated that the temperature difference is rather logarithmic than linear through the matrix under the flow condition studied.

The scatter observed is due to the internal structure of the regenerator matrix where can be clearly identify velocity gradient regions and low velocity (wake) regions where the heat transfer mechanism is negatively affected [17,18].

Fig. 9 shows the relationship between Nusselt number and Reynolds number for the numerical results obtained for a stacked and wound matrix configuration. Based on the numerical results obtained for different solid and working fluid properties, a specific Nusselt correlation equation is obtained for each matrix configuration. The specific Nusselt correlation equation for S110–63% is given by Eq. (12):

$$Nu = 1.91 + 0.17Re^{0.80} \quad (12)$$

And the specific Nusselt number correlation equation for W110–63% is given by Eq. (13):

$$Nu = 2.15 + 0.07Re^{0.88} \quad (13)$$

The Nusselt number correlation equations are used to define the heat transfer coefficient through a local thermal non-equilibrium porous media model for the full Stirling regenerator. The heat transfer coefficient is defined as:

$$h = \frac{Nu \cdot k_f}{d_h} \quad (14)$$

Although these coefficients can be experimentally obtained it is known that the experimental studies are limited and expensive due to the internal structures of the regenerator matrix. Therefore, the numerical approach presented is a cost effective tool compared to experimentation that can be used with confidence in the characterization of woven wire regenerator matrices to reduce experimental cost and improve internal geometry.

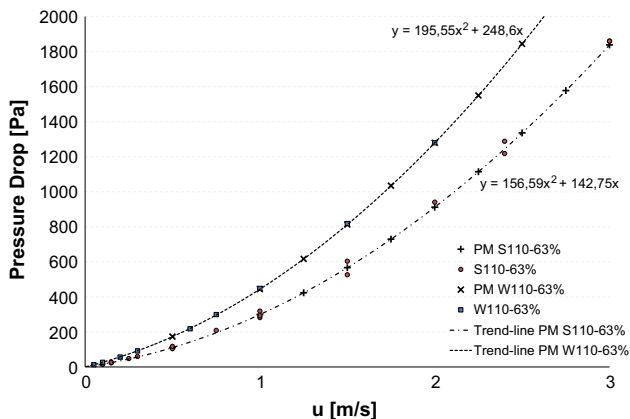


Fig. 11. Equivalent porous media pressure drop (Pa) vs. inlet velocity (m/s) trend-line for volumetric porosity of 63% at constant working fluid properties.

3. Computational principles

3.1. Governing equations for porous media

3.1.1. Momentum equations for porous media

The porous media is modelled by the addition of a source term to the momentum equation. The source term is composed of two parts: a viscous loss term (Darcy, the first term on the right-hand side of Eq. (15)), and an inertial loss term (the second term on the right-hand side of Eq. (15)).

$$S_i = - \left(\sum_{j=1}^3 D_{ij} \mu v_j + \sum_{j=1}^3 C_{ij} \frac{1}{2} \rho_f |v| v_j \right) \quad (15)$$

This momentum sink contributes to the pressure gradient in the porous cell, creating a pressure drop that is proportional to the fluid velocity (or velocity squared) in the cell. Eq. (16) recovers the case of simple homogeneous porous media:

$$S_i = - \left(\frac{\mu}{\alpha} v_i + C_2 \frac{1}{2} \rho_f |v| v_i \right) \quad (16)$$

Assuming an isotropic porous media and based on physical velocity, the momentum conservation equation is as follows:

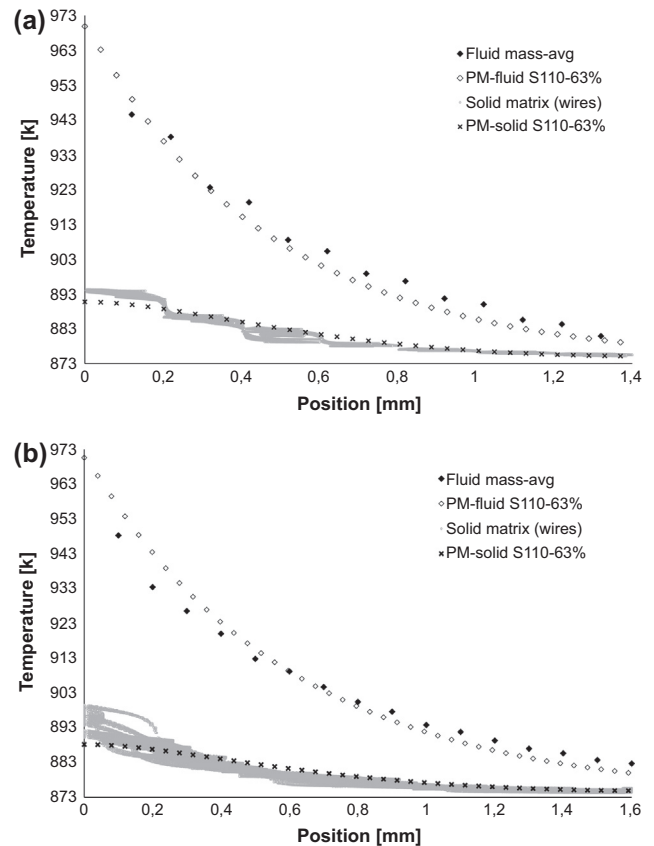


Fig. 12. Equivalent porous media, wire and working fluid temperature (K) vs. Position (mm) through woven wire matrix of 63% volumetric porosity for $Re \approx 65$ (1 m/s) at 0.02 s: (a) stacked and (b) wound.

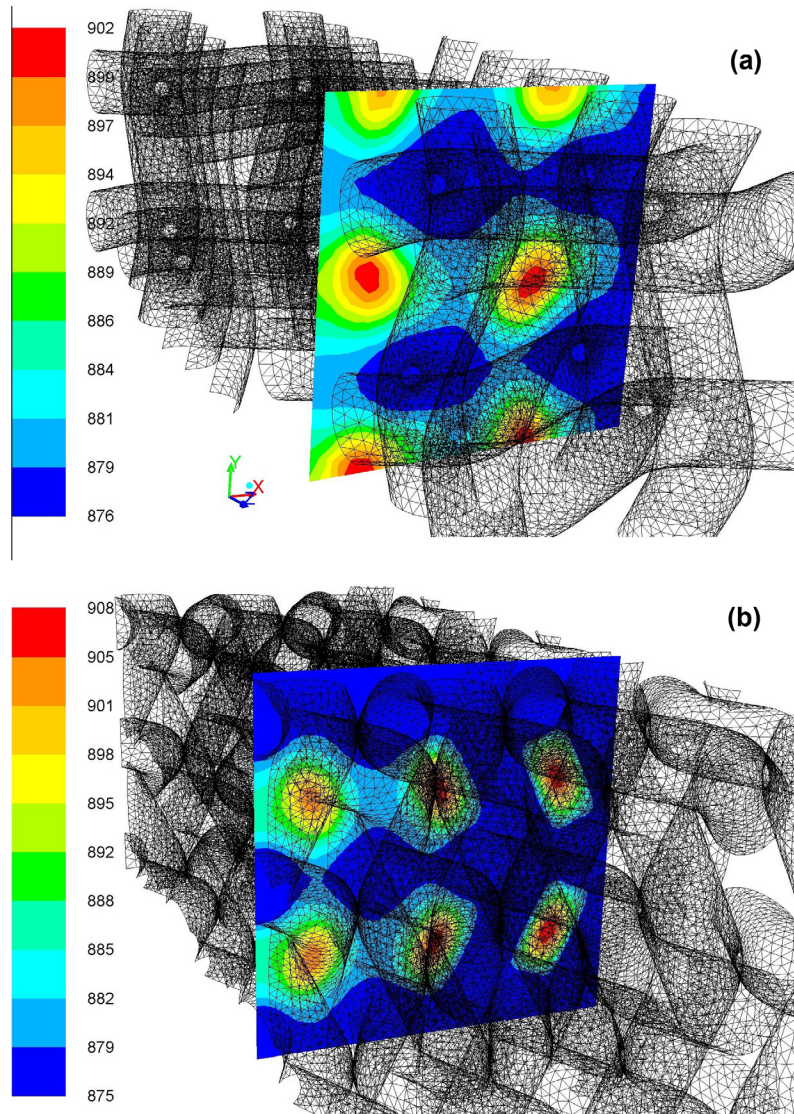


Fig. 13. Temperature contours (K) in a section plane 1 mm for 3-D detailed matrix of 63% volumetric porosity for $Re \approx 65$ (1 m/s) at 0.02 s: (a) stacked and (b) wound.

$$\frac{\partial(\gamma \rho_f \vec{v})}{\partial t} + \nabla \cdot (\gamma \rho_f \vec{v} \vec{v}) = -\gamma \nabla p + \nabla \cdot (\gamma \vec{\tau}) + \gamma \vec{B}_f - \left(\frac{\mu}{\alpha} + C_2 \frac{1}{2} \rho_f |\vec{v}| \right) \vec{v} \quad (17)$$

3.1.2. Energy equations for porous media

For the local thermal non-equilibrium porous media approach, a solid zone that is spatially coincident with the porous fluid zone is defined, and this solid zone only interacts with the fluid with regard to heat transfer. The conservation equations for energy are solved separately for the fluid and solid zones. The conservation equation solved for the fluid zone is:

$$\frac{\partial}{\partial t} (\gamma \rho_f E_f) + \frac{\partial}{\partial x_i} (v_i (\rho_f E_f + p)) = \frac{\partial}{\partial x_i} \left[\gamma k_f \frac{\partial T_f}{\partial x_i} + \tau_{ij} v_j - \left(\sum_i h_i J_i \right) \right] + S_f^h + h_{fs} A_{fs} (T_f - T_s) \quad (18)$$

and the conservation equation solved for the solid zone is:

$$\frac{\partial}{\partial t} ((1 - \gamma) \rho_s E_s) = \frac{\partial}{\partial x_i} \left((1 - \gamma) k_s \frac{\partial T_s}{\partial x_i} \right) + S_s^h - h_{fs} A_{fs} (T_f - T_s) \quad (19)$$

3.2. Numerical methodology

The fluid is considered to be viscous, incompressible, Newtonian and 2-D through the equivalent porous media zone using true (physical) velocity conditions under laminar flow assumption. The 3-D detailed geometry is relevant for the modelling of a woven wire regenerator matrix because the flow characteristic is intrinsically 3-D. However, modelling an equivalent isotropic porous media a simplified 2-D domain fully represents the detailed 3-D model. On the other hand, the 2-D domain is the optimal model for minimizing computational memory and time which is especially important under oscillating flow conditions.

The governing flow equations are discretized and solved sequentially using a finite volume method (FVM) based numerical flow solver [24] with a second order upwinding scheme for the discretization of the continuity, momentum and energy equations. The convergence criterion for all the velocity components and for the continuity is set to 10^{-6} , whereas for energy it is set to 10^{-8} for all simulations. The non-dimensional time step, $\tau = u_{\max} t / s$ is expressed as the product of the maximum velocity component at the inflow boundary and time elapsed divided by the volume cell size, in the porous media study the non-dimensional time step values are chosen as 1.

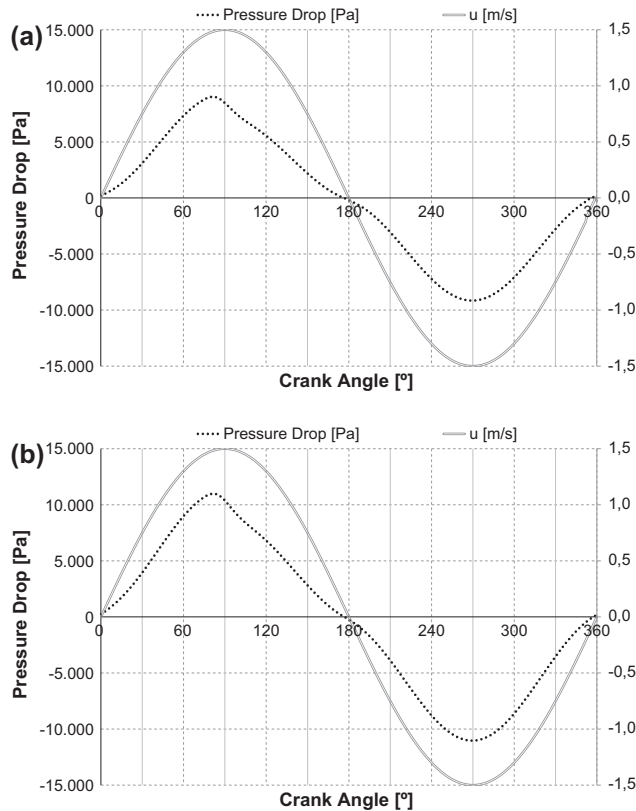


Fig. 14. Pressure drop (Pa) vs. crank angle (°) through: (a) stacked and (b) wound matrices.

3.2.1. Numerical methodology for equivalent porous media

The flow is unidirectional, unsteady and all fluid properties (including density, viscosity, specific heat, and conductivity) are assumed constant due to the small range of temperature used in each case (micro-scale model).

3.2.2. Numerical methodology for full regenerator as a porous media

For the simplified full regenerator solution, the flow is oscillating, unsteady, and the properties of the working fluid and the solid metal wire depend on the temperature. The working fluid is nitrogen and the metal wire matrix material is stainless steel.

3.3. Computational domain and boundary conditions

3.3.1. Computational domain and boundary conditions for equivalent porous media

Fig. 10 illustrates the 2-D domain of the flow of interest (geometry set-up) which includes a porous zone. The porous zone is equivalent to the detailed woven wire matrix previously characterized by the authors [17,18]. The rest of the domain remains the same and the boundary conditions and simulation options are not modified with respect to previous heat transfer studies for woven wire matrix, S110–63% and W110–63%.

The effect of the mesh resolution and time step size on the present flow is assessed through a grid independence study for three different mesh systems containing non-uniformly hexahedral grid cells and for three non-dimensional time step (from 0.1 to 1) at a Reynolds number of 200. In the temperature distribution values obtained from different mesh configurations and a non-dimensional time step, there is no major difference among the computed values through a porous media.

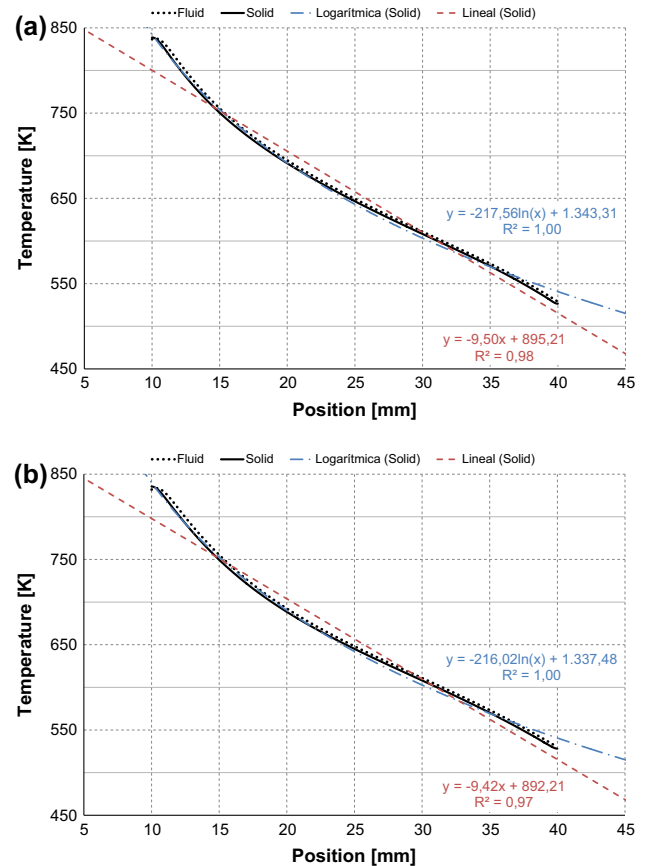


Fig. 15. Temperature profile (K) vs. Position (mm) through porous matrix for: (a) stacked and (b) wound.

The present simplified porous media model simulations are carried out by considering the following simplified boundary conditions:

1. *Inflow boundary*: Simplified uniform velocity inlet boundary conditions at constant temperature are used to define the fluid uniform velocity profile.
2. *Outflow boundary*: Pressure outlet boundary conditions are assigned to define the static (gauge) pressure by eliminating the reverse flow problem.
3. *Side boundary*: Free-slip symmetry flow boundary conditions at the two side boundaries of the computational domain are imposed.
4. *Porous media zone*: The settings and inputs for the porous media are the following: physical velocity formulation, viscous and the inertial resistance coefficients summarized in Table 1, isotropic porosity of 63%, interfacial area density (see Table 1) and constant heat transfer coefficient.

3.3.2. Computational domain and boundary conditions for full regenerator as a porous media

The idea is to model a simplified full size regenerator porous media under oscillating regeneration cycles. The full size regenerator matrix consists of the total length of a Stirling regenerator used in a micro-CHP reference engine (30 mm), and the full flow area in the regenerator is not modelled. The computational domain is constructed of hexahedral meshes and 200 time steps per cycle at 25 Hz is chosen.

The porous media model simulations are carried out by considering the following boundary conditions:

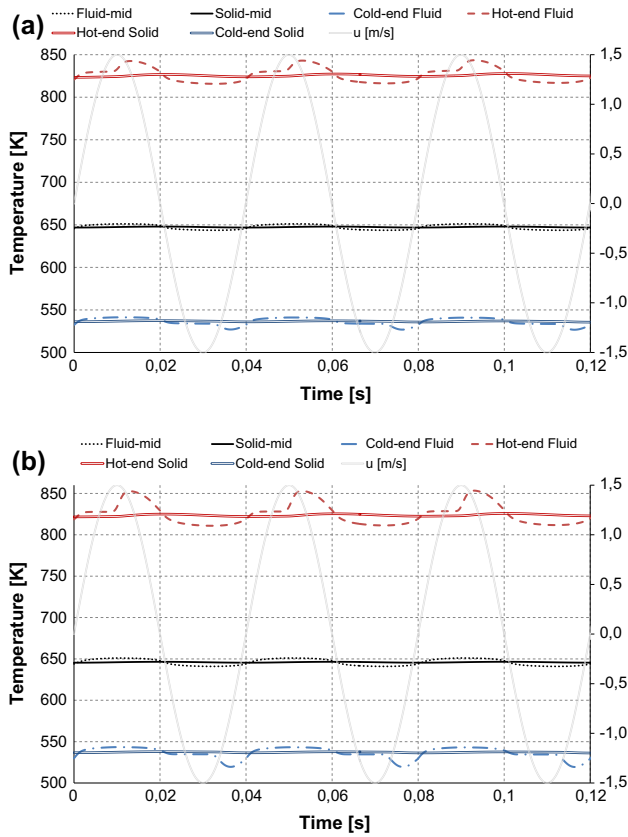


Fig. 16. Temperature fluctuations (K) in three regenerator sections vs. time (s) for: (a) stacked and (b) wound.

1. *Inflow boundary*: Oscillating inlet velocity boundary conditions at constant temperature (873 K, corresponding with the engine hot end) is used to define the fluid sinusoidal velocity profile defined with a user-defined-function (UDF) as follows:

$$u = u_0 \sin(2\pi ft) \quad (20)$$

where u_0 is the amplitude of the oscillating inlet velocity (1.5 m/s) and f is the engine frequency (25 Hz).

2. *Outflow boundary*: Pressure outlet boundary conditions at constant temperature (423 K, corresponding with the engine cold end). The working pressure is constant at 26 bar.
3. *Side boundary*: Free-slip symmetry flow boundary conditions at the two side boundaries of the computational domain are imposed. The normal velocity components and the normal gradients of all velocity components are assumed to have a zero value.
4. *Porous media zone*: The settings and inputs for the porous media are the following: physical velocity formulation, viscous and the inertial resistance coefficients summarized in Table 1, isotropic porosity of 63%, constant interfacial area density (see Table 1) and variable heat transfer coefficient. The variable heat transfer coefficient of the porous medium is defined through a UDF using Eqs. (8), (12), (13) and (14).

4. Results and discussion

4.1. Numerical validation for equivalent porous media

The results obtained from the numerical simulations, which are performed for the equivalent 2-D porous media of the stacked

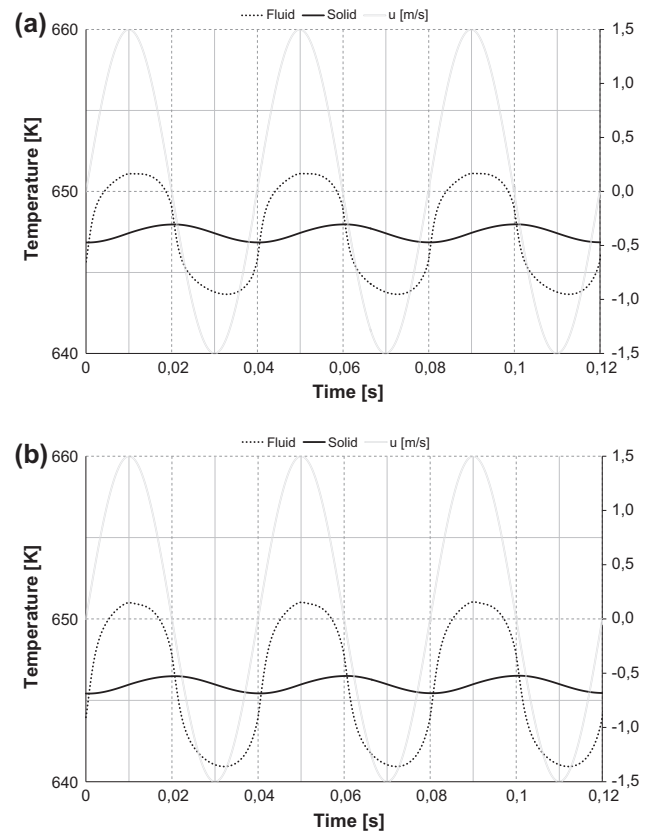


Fig. 17. Temperature fluctuations (K) in the mid-section vs. time (s) for: (a) stacked and (b) wound.

woven wire matrix S110–63% and of the wound woven wire matrix W110–63% are described for the detailed matrix configuration.

Fig. 11 shows the pressure drop results as a function of the inlet velocity for S110–63% and W110–63% equivalent porous media. The figure also shows the numerical results obtained from the detailed regenerator matrix simulation solutions for both configurations. Excellent agreement between the results of 3-D regenerator matrix and the equivalent 2-D porous media are obtained using the viscous and inertia resistance coefficients derived from the detailed 3-D regenerator matrix summarized in Table 1.

Fig. 12 shows the numerical results for fluid and solid temperature distribution as a function of the length through the equivalent porous media. Fig. 12a compares the results for the detailed stacked woven wire matrix against its equivalent fluid and solid porous media, and Fig. 12b for the wound woven wire matrix. The fluid temperature distribution for the detailed matrix corresponds to the mass-average temperature. It is observed from the figure that there is a good agreement between the detailed matrix temperatures and the equivalent porous media. It is noteworthy here that the temperature distribution through the isotropic porous media is smooth and the present model cannot reproduce the stepped behaviour in the solid wire matrix solution.

Fig. 13 represents the temperature contours in a cutting plane at 1 mm through both detailed 3-D regenerator matrices. The figure signifies that the temperature gradient in the fluid is significant for both configurations as previously observed in Fig. 6. Through an isotropic porous media in a transversal cutting plane, the temperature of the fluid and solid are constant. For this reason the differences observed in the results of the equivalent porous media and the 3-D model are acceptable considering the temperature gradient behaviour observed in the figure. Therefore, it is believed that the results verify the suitability of the local thermal non-equilibrium

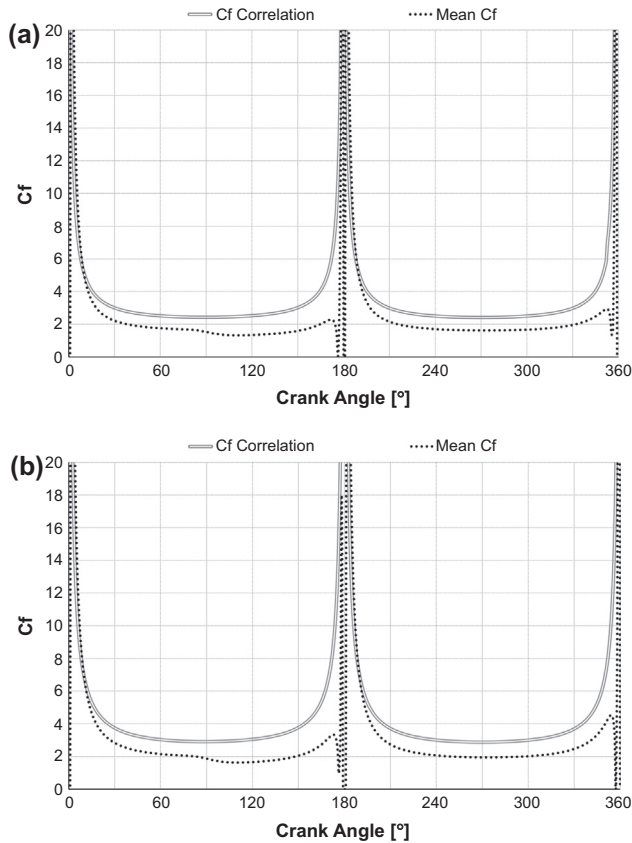


Fig. 18. Porous matrix friction factor and specific friction factor correlation equation vs. crank angle (°) for: (a) stacked and (b) wound matrices.

porous media model as an equivalent of the detailed 3-D regenerator matrices for pressure drop and heat transfer phenomena.

4.2. Numerical validation for full regenerator as a porous media

In this section, the numerical simulations for the simplified full length 2-D equivalent porous media of the stacked and of the wound woven wire matrix are performed under oscillating flow conditions. In these models under oscillating regeneration cycles the desired solution is the quasi-steady solution, when the results for two consecutive cycles are identical. The results present and discussed here are obtained in a quasi-steady state for the working pressure of 26 bar and the flow oscillating frequency of 25 Hz.

Fig. 14 shows the variations of the computed pressure drop through the porous media matrix for both configurations during a complete quasi-steady cycle. It is noted that for the frequency studied, the pressure drop varies sinusoidal and a small phase lag during the first half of the cycle (hot-blow) with respect to the inlet fluid velocity is observed. The cycle-averaged pressure drop for the S110–63% equivalent regenerator porous media is 4700 Pa and for the W110–63% equivalent regenerator porous media is 5780 Pa.

The temperature profiles through the regenerator porous matrix for the fluid and solid are presented in Fig. 15. A logarithmic and linear trend-line are both included into the figure as reference lines. It can be found that the temperature profile through the regenerator can be better fitted to a logarithmic curve (regression coefficient of about 1), rather than linear line. Therefore, the logarithmic mean temperature difference between fluid and solid is used for the present heat transfer calculations.

Fig. 16 illustrates the temperature fluctuations in three sections of the regenerator porous media matrix (mid-plane, hot-end (1 mm) and cold-end (1 mm)) during three complete quasi-steady

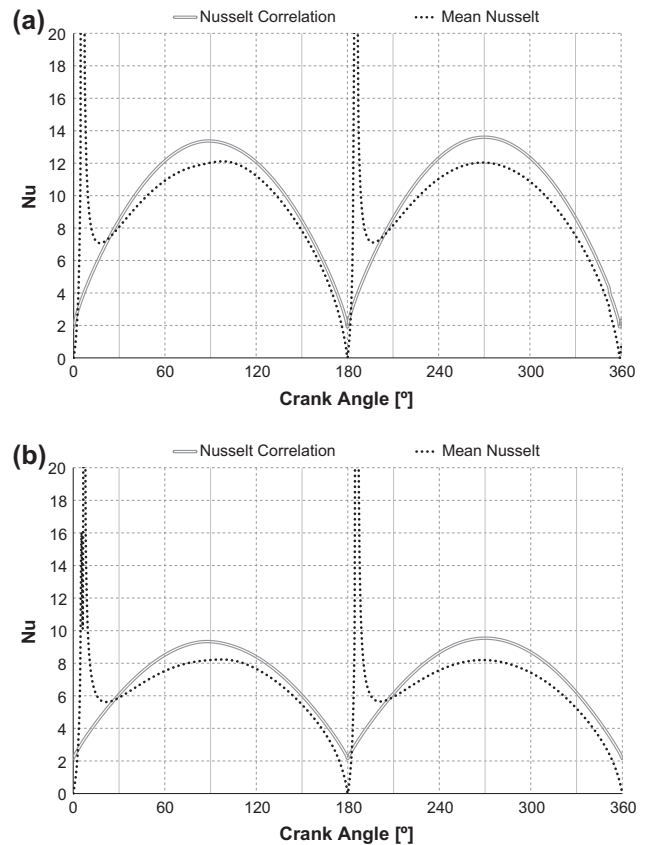


Fig. 19. Porous matrix Nusselt number and Specific Nusselt number correlation equation vs. crank angle (°) for: (a) stacked and (b) wound matrices.

cycles. It is observed that in different axial sections the solid matrix temperature corresponds to the mean fluid matrix temperature. On the other hand, both fluid and solid temperatures change periodically with the oscillating flow and the variations of the fluid temperature is larger than the solid temperature (almost constant).

The temperature fluctuations in the mid-section for stacked and wound matrix configurations are also illustrated in Fig. 17. A phase lag is clearly identified for the matrix temperature due to the heat transfer process. Basically, the fluid temperature is higher during the first half of the oscillating cycle, while at the second half, the solid temperature is higher.

Fig. 18 represents a comparison between the friction factor numerically obtained from the 3-D detailed matrix (Eqs. (10) and (11)) at local instantaneous Re number with the friction factor obtained through a whole porous media under oscillating flow conditions at average Re number. The figure clearly shows that the profile of the computed local instantaneous friction factor corresponds well with that of average friction factor.

Fig. 19 illustrates the Nusselt number variation with respect to the crank angle for the specific numerically obtained correlation from the 3-D detailed matrix (Eqs. (12) and (13)) with the average Nusselt number obtained in the present study through a porous media under oscillating flow conditions. The Nusselt number peaks are due to the very small temperatures difference between the fluid and the solid matrix during direction change under oscillating flow. The mean Nusselt number obtained through the porous media under oscillating flow conditions characterize the heat transfer performance of the 3-D detailed regenerator matrix. Comparing the temperatures results shown in Fig. 16, there is not a significant variation in the temperatures obtained for both configurations. However, if the thermal efficiency is calculated a noticeable difference may be obtained.

5. Conclusions

The present study proposes a stepwise numerical procedure for the derivation of the flow resistance coefficients and thermal non-equilibrium heat transfer coefficient for porous media of Stirling regenerator under unidirectional flow conditions. The accuracy of the equivalent porous media modelling technique is verified against the validated numerical results obtained for the detailed 3-D woven wire matrices. The methodology is successfully adopted for the flow and heat transfer characterization of two different woven wire matrix configurations, stacked and wound with 63% of volumetric porosity.

It is remarkable that for specific matrix configuration, the friction factor correlation equation form that better fits with the numerical results is a two-parameter Ergun form. In the case of a general correlation equation a third parameter could be necessary to better fit the curve at high Reynolds numbers [17,19]. On the other hand, the heat transfer results demonstrate that the trend-line obtained for the temperature profiles through the regenerator porous matrix for the fluid and solid can be better fitted to a logarithmic curve rather than linear under oscillating flow conditions.

The present results suggest that the pressure drop and heat transfer performance of the different configuration matrices can be evaluated and compared as well as the thermal efficiency or the figure of merit. In the characterization of a stacked and a wound woven wire matrices is observed a better overall performance in terms of pressure drop and heat transfer for the stacked configuration.

It is believed that the numerical characterization methodology presented is advantageous for the design and optimization of the regenerator matrix under diverse working conditions (oscillating frequency, working pressure, working gas, matrix material, etc.) and useful for the multi-D Stirling engine models.

References

- [1] Ibrahim M, et al. CFD modelling of free piston stirling engine [Book]. Ohio, NASA/TM-2001-211132; 2001.
- [2] Wilson SD, Dyson RW, Tew RC, Ibrahim M. Multi-D CFD modelling of a free-piston stirling convertor at NASA Glenn [Report] = AIAA-2004-5673. [s.l.], NASA/TM-2004-213351; 2004.
- [3] Dyson RW, Wilson SD, Tew RC, Demko R. On the need for multidimensional stirling simulation. NASA/TM-2005-213975. AIAA-2005-5557.
- [4] Tew R, et al. An initial non-equilibrium porous-media model for CFD simulation of stirling regenerators [Book]. Ohio: NASA/TM-2006-214391; 2006.
- [5] Dyson RW, Geng SM, Tew RC, Adelino M. Towards fully three-dimensional virtual Stirling convertors for multi-physics analysis and optimization. *Eng Appl Comput Fluid Mech* 2008;2(1):95–118.
- [6] Avdeychik V, Evstigneev N, Shmelev A. Application of the 3-dimensional CFD model to a geometric parameters optimization and an analyze of dimensional aspects of the Stirling engine. In: Conference ISEC international stirling engine committee, 2009.
- [7] Kraitong Kwanchai, Mahkamov, Khamid. Thermodynamic and CFD modelling of low-temperature difference stirling engines. In: Conference ISEC international Stirling engine committee, 2009.
- [8] Ibrahim M, et al. Improving performance of the stirling converter: redesign of the regenerator with experiments, computation and moder fabrication techniques. Cleveland State University; 2001.
- [9] Park JW, Ruch D, Wirtz RA. Thermal/fluid characteristics of isotropic plain-weave screen laminates as heat exchange surfaces. Reno: Am Inst Aeronaut Astronaut 2002;AIAA 2002-0208.
- [10] Kim Sung-Min. Numerical Investigation on Laminar Pulsating Flow Through Porous Media [Report]. [s.l.] Georgia Institute of Technology; 2008.
- [11] Tao YB, Liu YW, Gao F, Chen XY, He YL. Numerical analysis on pressure drop and heat transfer performance of mesh regenerators used in cryocoolers. *Cryogenics* 2009;49:497–503.
- [12] Landrum EC, Conrad TJ, Ghiaasiaan SM, Kirkconnell CS. Effect of pressure on hydrodynamic parameters of several PTR regenerator fillers in Axial steady flow [conference]//International cryocooler conference. vol. 15, 2009.
- [13] Conrad TJ, et al. Anisotropic hydrodynamic parameters of regenerator materials suitable for miniature cryocoolers [Conference]//International cryocooler conference; 2009.
- [14] Nair Anjan R, Krishnakumar K. Numerical modeling of single blow transient testing of a wire screen mesh heat exchanger [conference]//10th National conference on technological trends; 2009.
- [15] Forooghi P, Abkar M, Saffar-Avval M. Steady and unsteady heat transfer in a channel partially filled with porous media uner thermal non-equilibrium condition. *Transp Porous Media* 2010.
- [16] Pathak Mihir Gaurang. Periodic flow physics in porous media of regenerative cryocoolers. In: [Report]: dissertation. [s.l.] Georgia Institute of Technology; 2013.
- [17] Costa SC, Barrutia H, Esnaola JA, Tutar M. Numerical study of the pressure drop phenomena in wound woven wire matrix of a Stirling regenerator. *Energy Convers Manage* 2013;67:57–65.
- [18] Costa SC, Barrutia H, Esnaola JA, Tutar M. Numerical study of the heat transfer in wound woven wire matrix of a Stirling regenerator. *Energy Convers Manage* 2014;79:255–64.
- [19] Gedeon D, Wood JG. Oscillating-flow regenerator test rig: hardware and theory with derived correlations for screens and felts. NASA CR-198442; 1996.
- [20] Ergun S. Fluid flow through packed columns. *Chem Eng Prog* 1952;48:89–94.
- [21] Miyabe H, Takahashi S, Hamaguchi K. An approach to the design of Stirling Engine Regenerator matrix using packs of wire gauzes. *Proc IECEC* 1982;17:1839–44.
- [22] Tanaka M, Yamashita I, Chisaka F. Flow and heat transfer characteristics of the stirling engine regenerator in an oscillating flow. *JSME Int J* 1990;33:283–9.
- [23] Thomas B, Pittman D. Update on the evaluation of different correlations for the flow friction factor and heat transfer of stirling engine regenerators. Am Inst Aeronaut Astronaut 2000.
- [24] Fluent Dynamics Software. ANSYS FLUENT. Lebanon, NT 6.1.7601; 2010.